Mathematics as a Handicraft – Gestures and Actions of Young Learners while Working on Diagrams

Lara Billion, Melanie Huth & Rose Vogel

Theoretical Background – Diagrammatic Reasoning

This poster focusses on actions as well as gestures of learners while the use of self-generated mathematical diagrams. Therefore, a semiotic perspective on learning mathematics in the sense of C. S. Peirce is considered (CP, 1931-1935). According to this approach, mathematics is seen as a visible activity in which "the very objects of interest, of learning and communication are now perceivable and communicable if math is understood (primarily and initially) as a social practice with, on, about, and through diagrams." (Dörfler, 2006, p. 105). Learning mathematics can be understood as a handicraft of using signs and diagrams (Dörfler, 2006). The concept of diagrams describes rule-based fixations of different kinds: Use material and generate a specific order of this material; write something down or even use a gesture to generate a (mathematical) diagram. Reasoning with, on, and about diagrams in different modes is seen as a central activity in mathematics learning. It consists of constructing a diagram, exploring relations within this diagram and reflecting on the results of manipulations on the diagram. This use of diagrams in mathematical interactions is seen as a highly creative process, wherein learners earn insights in mathematical structures and get access to a deeper mathematical understanding (Dörfler, 2006). Former research on learner-generated diagrams show that diagrammatic reasoning takes place in a multimodal way. By material-based actions and gestures learners create diagrams and represent relevant diagrammatic rules (e.g. Billion & Vogel, 2021; Huth, 2022; Vogel, 2017). Huth (2022) reconstructs different gestural functions in learning mathematics, ranging from discourse-structuring to the gestural creation of diagrams. Billion (2021) can find evidence that diagrammatic interpretations of learners can be reconstructed by actions on materials. Vogel and Huth (2020) reconstruct three different interfaces of gestures and actions in mathematical interactions of learners: interfaces of *chronology*, *semantic meaning,* and *function* (Vogel & Huth, 2020).

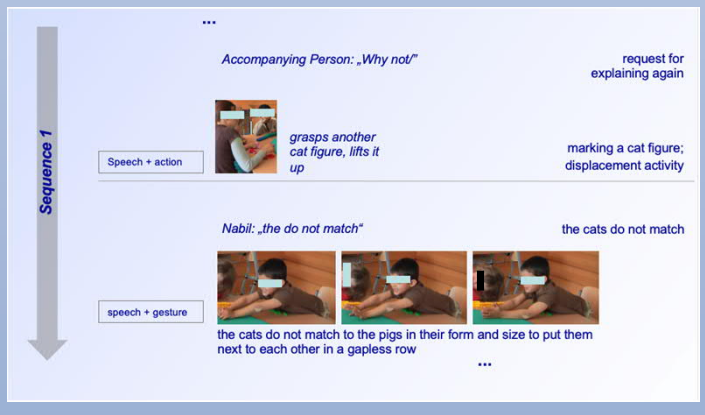
The Example

An example of the longitudinal designed erStMaL-project is analysed (early Steps in Mathematics Learning, n = 144 kindergartners; IDeA-Centre, Frankfurt/Main): Four kindergarteners are occupied with a huge set of wooden animal figures in different sizes, forms, and colours, which can be e.g., sorted, counted or used to generate patterns (Billion, Huth & Möller, 2020). The focus is on one child, Nabil (second-language learner, German) interacting with the accompanying person:

1. In which way do the diagrammatic interpretations of the interactors (Nabil and accompanying person) differ?
2. Do the same relations come to the fore in the gestural and action-based utterances of the participants?

In the analyzed situation, Nabil is sitting opposite the person accompanying him and cannot reach the crowd of cats and pigs lying in front of her. Nabil tries four times to refer in gestures and speech to the fact that something is not the same. His pointing gestures are directed to the material arrangement in front of the accompanying person, which can be interpreted as a diagram. She takes up Nabil's gestures to the diagram by repeatedly grasping different parts of the material arrangement.

After the accompanying person realizes that Nabil means the cats, she interprets Nabil's gestures and speech in such a way that he thinks the cats do not fit in with the crowd in front of her. And then she asks again:



Later Nabil refers to the different size of the cats and pigs. Before the wooden animals were only sorted species-wise in a row. Thus, Nabil recognizes a new mathematical relationship on the diagram of pigs and cats. Through Nabil's new mathematical idea, a new interpretation of the diagram in terms of ordering by size becomes possible. Nabil instructs the accompanying person, who has access to the diagram in front of her, to act on the diagram according to his intended diagram manipulations.

Conclusion

At the beginning of the interaction between Nabil and the accompanying person, the two interactors have a different interpretation of the diagram. Through the interplay of gestures and actions, the interpretations continue to converge. Together they produce a new mathematical relationship of the diagram in a multimodal way. In this example, it can be shown how gestural manipulations of a learner can be translated by the accompanying person into acting manipulations.

Billion, L. (2021). Reconstruction of the Interpretation of Geometric Diagrams of Primary School Children Based on Actions on Various Materials – A Semiotic Perspective on Actions. International Electronic Journal of Mathematics Education, 16 (3). https://doi.org/10.29333/iejme/11068
Billion, L., Huth, M., & Möller, V. (2020). Die Schweine sind zu klein. Die Rekonstruktion mathematischer Konzepte und Gesten in einer Spiel- und Erkundungssituation im Kindergarten. In M. Beck, L. Billion, M. Fetzer, M. Huth, V. Möller, & A. Vogler (Eds.), Multiperspektivische Analysen von Lehr-Lernprozessen. Multimodale, mathematikdidaktische, digitale und konzeptionelle Ansätze in verschiedenen Bildungskontexten. Festschrift für Prof.in Dr. Rose Vogel (pp. 11-36). Münster: Waxmann.
Billion, L. & Vogel, R. (2021) Material as an Impulse for Mathematical Actions in Primary School – A Semiotic Perspective on a Geometric Example. In M. Inprasitha, N. Changsri & N. Boonsena (Eds), Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education (Vol.2) (pp. 93–101). Khon Kaen, Thailand: PME.
Dörfler, W. (2006). Inscriptions as Objects of Mathematical Activities. In J. Maaz & w. Schlögelmann (Eds.), New Mathematics Education Research and Practice (pp. 97–111). Rotterdam: Sense Publisher.
Huth, M. (2022). Handmade diagrams – Learners doing math by using gestures. 12th Congress of the European Society for Research in Mathematics Education (CERME).
Peirce, C. S. (CP). Collected Papers of Charles Sanders Peirce (Volumes I-VI, ed. by Charles Hartshorne and Paul Weiss, 1931-1935, Volumes VII-VIII, ed. by Arthur W. Burks, 1958, quotations according to volume and paragraph). Cambridge, Mass.: Harvard UP.
Vogel, R. (2017). Diagrammatischer Charakter von Handlungen an Objekten in mathematischen Spiel- und Erkundungssituationen. In U. Kortenkamp & A. Kuzle (Eds.), Beiträge zum Mathematikunterricht 2017 (pp. 993-996). Münster: WTM-Verlag.
Vogel, R., & Huth, M. (2020). Modusschnittstellen in mathematischen Lernprozessen. Handlungen am Material und Gesten als diagrammatische Tätigkeit. In G. Kadunz (Ed.), Zeichen und Sprache im Mathematikunterricht – Semiotik in Theorie und Praxis. Springer.